AP Calculus AB
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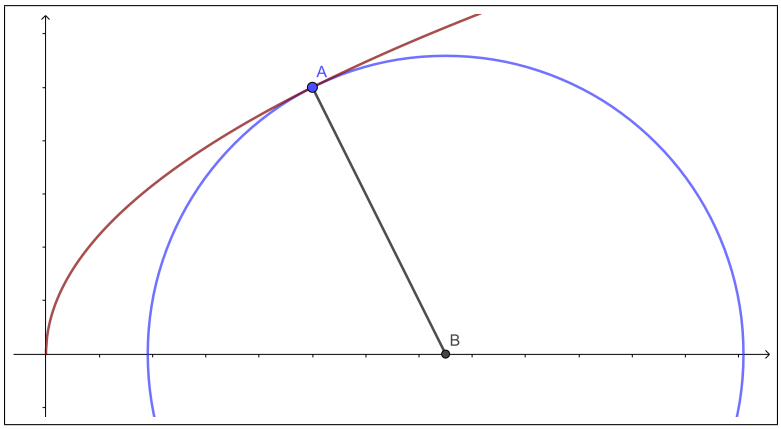
Activity 0820
Tuesday, August 20, 2024

Name:

Problem 1. Let $f(x) = \sqrt{x}$. Then the point $(1, 1)$ is a point on the graph of f. There is a unique line which passes through $(1, 1)$ and otherwise lies above the graph of f. This is called the *tangent line*.



Descartes found the slope of this line by first finding the unique circle, centered on the x-axis, which is tangent to the graph of f. Let $(h, 0)$ be the center of this circle. In the diagram above, point $A = (1, 1)$ is the point of tangency, and point $B = (h, 0)$ is the center of the circle.

(a) Find the distance r from A to B, written in terms of h.

(b) Write the equation of the circle centered at B with radius r.

(c) Plug $y = \sqrt{x}$ into the equation of the circle. You now have a quadratic equation in variable x and constant h. Put this equation in standard form $ax^2 + bx + c = 0$.

(d) Write the a, b, and c you found in part **(c)**. Plug a, b, and c into the discriminant $b^2 - 4ac$.

(e) We want the discriminant to be zero (why?). Set $b^2 - 4ac = 0$ and solve for h.

(f) Find the slope of the line through A and B.

(g) Find the slope of the tangent line.